

On a grounded stators ESL cell with a bipolar diaphragm

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Development of grounded stators and bipolar diaphragm concept starting from the analysis of the equivalent circuits of known ESL driving methods .

Prologue

As is well recognized, the force developed between the two plates of area A in a parallel-plates capacitor with a gap d maintained charged at a voltage V is given by

$$F = \frac{\epsilon_0 A V^2}{2d^2} \quad (1) [1]$$

where $\epsilon_0 = 8.854 \cdot 10^{-12}$ F/m is the absolute dielectric permittivity (dielectric constant) of the air, which is very close to that of the vacuum.

The square on the voltage in this equation tells that the force F developed by a parallel-plates capacitor can not be used as an audio source as the relationship with the voltage is not linear, but also tells that the force F is always attractive towards the plates.

As a classic ESL cell can be seen as two parallel plates capacitors with one common plate (the diaphragm), the force F_{diaph} developed on this common plate is simply given by the difference between the attractive forces F_1 between the first stator and the diaphragm and F_2 between the diaphragm and the second stator

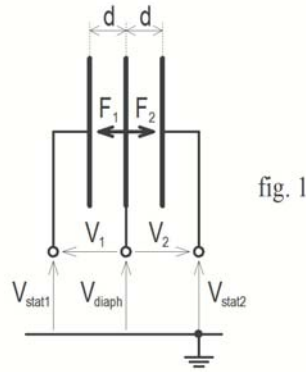
$$F_{\text{diaph}} = F_1 - F_2 = \frac{\epsilon_0 A V_1^2}{2(d-x)^2} - \frac{\epsilon_0 A V_2^2}{2(d+x)^2} \quad (2) [2]$$

where x is the displacement of the diaphragm due to its motion towards the highest square of the voltages V_1 and V_2 , being in this case $V_1^2 > V_2^2$.

The voltages V_1 and V_2 applied between the plates can be written as the voltage differences between the voltages on the three ESL cell terminals and a common reference, for example the ground potential (fig. 1)

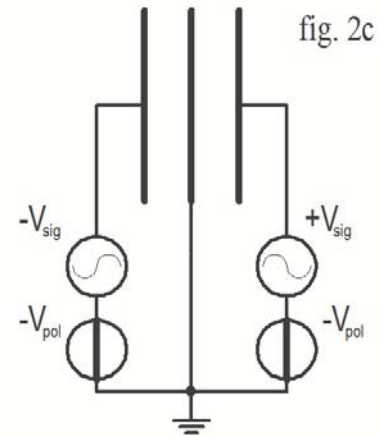
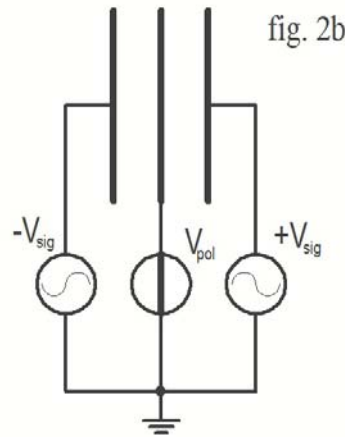
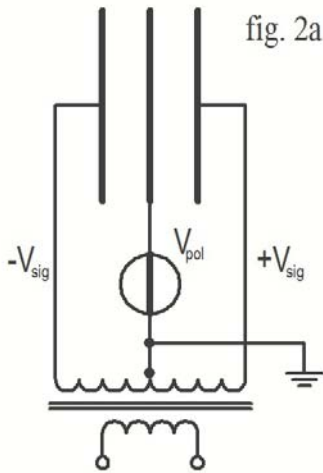
$$V_1 = V_{stat1} - V_{diaph} \quad (3)$$

$$V_2 = V_{stat2} - V_{diaph}$$



Grounding the diaphragm in known driving circuits

When the ESL cell is driven by the classic “push-pull” method with a center tap transformer that provides two 180° out-of-phase audio signals to the stators and a fixed high voltage bias generator tied to the diaphragm as in fig. 2a and in its equivalent diagram 2b



we have $V_{stat1} = -V_{sig}$ $V_{diaph} = V_{pol}$ $V_{stat2} = +V_{sig}$

and thus $V_1 = V_{stat1} - V_{diaph} = -V_{sig} - V_{pol}$ (4)

$V_2 = V_{stat2} - V_{diaph} = +V_{sig} - V_{pol}$

but the same equations (4) can be obtained also by the equivalent circuit of fig.2c in which the bias generator V_{pol} has been duplicated and moved in series to the $\pm V_{sig}$ generators while the diaphragm has been tied to the ground.

If the CT-transformer is replaced by an SRPP (or a cascode) linear tube amplifier with differential outputs (fig.3) provided by a single power supply rail like in the Acoustat Servo-Charge Amplifier [3] or by other more modern similar linear circuit based on semiconductors, we have

$$V_{stat1} = -V_{sig} + V_Q \quad V_{diaph} = V_{pol} \quad V_{stat2} = +V_{sig} + V_Q$$

where $V_Q \neq 0$ is the quiescent point voltage at the amplifier outputs in case of DC coupling between the outputs and the stators and $V_Q = 0$ in case of AC coupling.

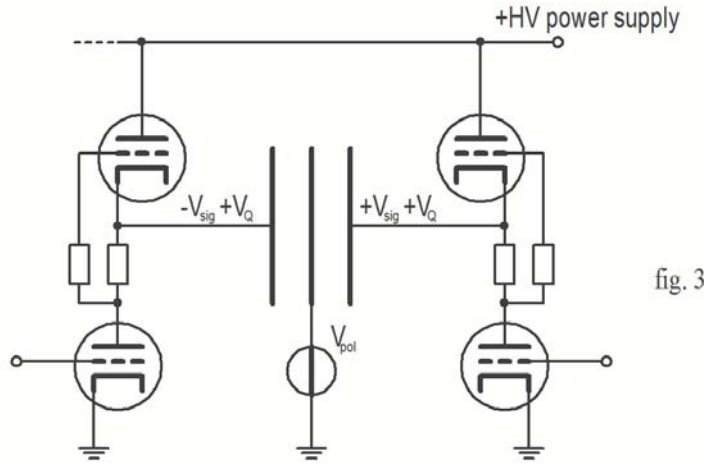


fig. 3

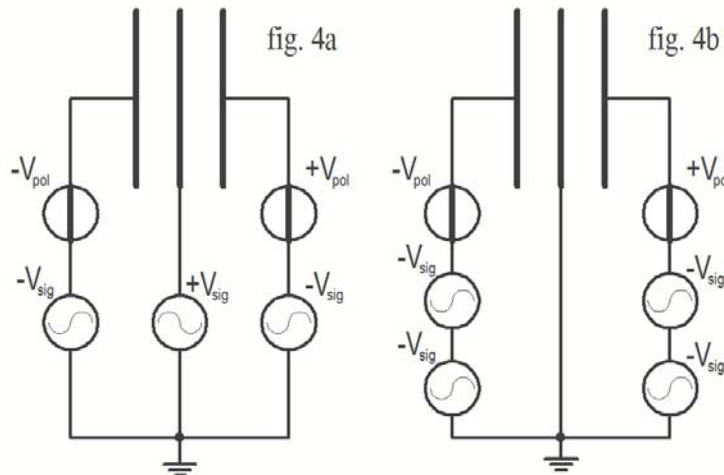
In this kind of circuit

$$V_1 = V_{stat1} - V_{diaph} = -V_{sig} + V_Q - V_{pol} \quad (5)$$

$$V_2 = V_{stat2} - V_{diaph} = +V_{sig} + V_Q - V_{pol}$$

that corresponds to the equivalent circuit of fig.2c with a V_Q generator added to both branches of the stators.

The third ESL cell driving method is the Beveridge's circuit, which is also based on a differential output SRPP tube amplifier but with a bipolar HV bias; its equivalent diagram is depicted in fig.4a



where

$$V_{stat1} = -V_{pol} - V_{sig} \quad V_{diaph} = +V_{sig} \quad V_{stat2} = +V_{pol} - V_{sig}$$

giving

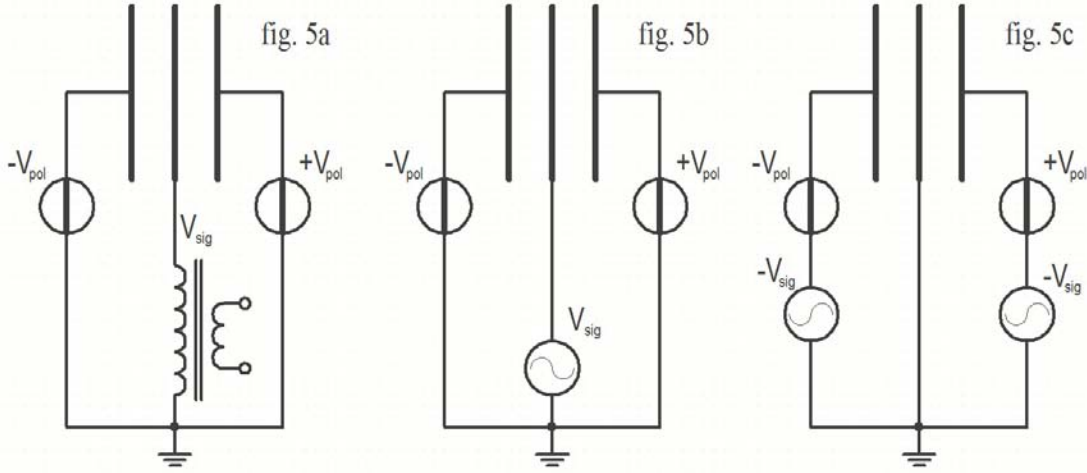
$$V_1 = V_{stat1} - V_{diaph} = -V_{pol} - V_{sig} - (+V_{sig}) = -V_{pol} - 2V_{sig}$$

$$V_2 = V_{stat2} - V_{diaph} = +V_{pol} - V_{sig} - (+V_{sig}) = +V_{pol} - 2V_{sig} \quad (6)$$

that are the same equations describing the grounded diaphragm equivalent circuit of fig.4c .

A further driving method has been proposed by Final Sound Solutions in their white paper [4]: the “Inverted ESL” is a circuit (fig. 5a) which equivalent diagram (fig.5b) is similar to the Beveridge's one, with a common mode V_{sig} driving the diaphragm and the stators being biased at HV of opposite sign

$$V_{stat1} = -V_{pol} \quad V_{diaph} = V_{sig} \quad V_{stat2} = +V_{pol}$$



In their circuit we have

$$V_1 = V_{stat1} - V_{diaph} = -V_{pol} - V_{sig}$$

$$V_2 = V_{stat2} - V_{diaph} = +V_{pol} - V_{sig}$$

(7)

that is again equivalent to the grounded diaphragm circuit of fig.5c .

We can summarize the above analysis on a table (tab.1)

CT – trans. (HV bias can be $-V_{pol}$ or $+V_{pol}$)	$V_1 = -V_{sig} \pm V_{pol}$ $V_2 = +V_{sig} \pm V_{pol}$	$V_1^2 = V_{sig}^2 \mp 2V_{sig} V_{pol} + V_{pol}^2$ $V_2^2 = V_{sig}^2 \pm 2V_{sig} V_{pol} + V_{pol}^2$
SRPP (DC coupled, $V_Q=0$ if AC coupled)	$V_1 = -V_{sig} + (V_Q - V_{pol})$ $V_1 = +V_{sig} + (V_Q - V_{pol})$	$V_1^2 = V_{sig}^2 - 2V_{sig}(V_Q - V_{pol}) + (V_Q - V_{pol})^2$ $V_2^2 = V_{sig}^2 + 2V_{sig}(V_Q - V_{pol}) + (V_Q - V_{pol})^2$
Beveridge	$V_1 = -2V_{sig} - V_{pol}$ $V_2 = -2V_{sig} + V_{pol}$	$V_1^2 = 4V_{sig}^2 + 4V_{sig} V_{pol} + V_{pol}^2$ $V_2^2 = 4V_{sig}^2 - 4V_{sig} V_{pol} + V_{pol}^2$
Final Sound Solutions	$V_1 = -V_{sig} - V_{pol}$ $V_2 = -V_{sig} + V_{pol}$	$V_1^2 = V_{sig}^2 + 2V_{sig} V_{pol} + V_{pol}^2$ $V_2^2 = V_{sig}^2 - 2V_{sig} V_{pol} + V_{pol}^2$

In the above table it can be seen that all the equations of V_1^2 and V_2^2 have the square terms with (obviously) the positive sign, while the cross terms $V_{sig}V_{pol}$ has always opposite signs, so that in

$$F_{diaph} = F_1 - F_2$$

the squared terms are cancelled while the cross terms are added and this gives the linear relationship between V_{sig} and F_{diaph} .

This happens because in the CT-transformer and in the SRPP circuits the V_{pol} generators have the same signs while the V_{sig} generators are 180° out-of-phase and because in the Beveridge and in the Final Sound Solution circuits the arrangement is reversed: V_{sig} are in-phase and V_{pol} have opposite signs, as summarized in the following table (tab.2)

	V_{sig} out-of-phase	V_{sig} in-phase
V_{pol} same signs	CT-Transf. SRPP	
V_{pol} opposite signs		Beveridge Final Sound Sol.

The remaining two combinations are never used as driving method.

With V_{sig} out-of-phase and V_{pol} of opposite signs we will have

$$\begin{aligned} V_1 &= +V_{sig} \pm V_{pol} \\ V_2 &= -V_{sig} \mp V_{pol} \end{aligned} \quad (8)$$

$$\text{thus } V_1^2 = V_{sig}^2 \pm 2V_{sig}V_{pol} + V_{pol}^2 \quad \text{and} \quad V_2^2 = V_{sig}^2 \pm 2V_{sig}V_{pol} + V_{pol}^2 \quad (9)$$

means in $F_{diaph} = F_1 - F_2$ all the terms of V_1^2 cancel those of V_2^2 so no any sound is developed by F_{diaph} .

The same happens with V_{sig} in-phase and V_{pol} of same signs

$$\begin{aligned} V_1 &= \pm V_{sig} \pm V_{pol} \\ V_2 &= \pm V_{sig} \pm V_{pol} \end{aligned} \quad (10)$$

$$\text{giving } V_1^2 = V_2^2 = V_{sig}^2 + 2V_{sig}V_{pol} + V_{pol}^2 \quad (11)$$

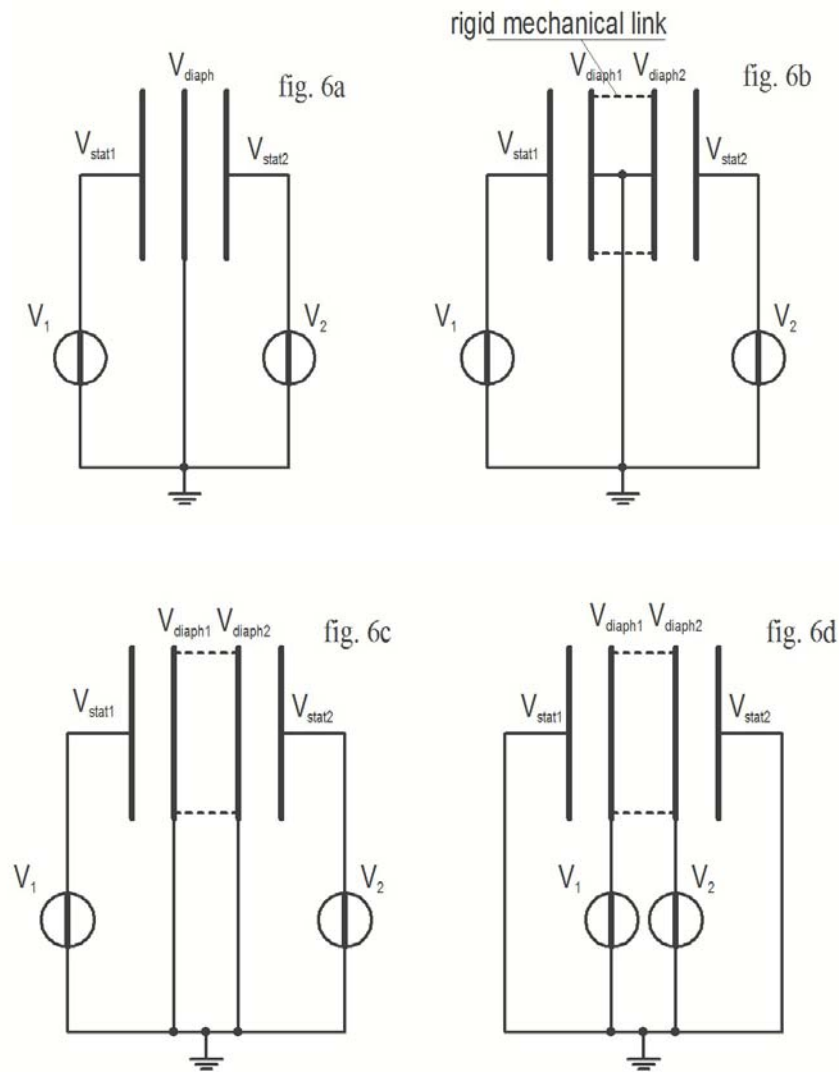
thus canceling again F_{diaph} .

Development of a grounded stators ESL cell

In the previous paragraph it was demonstrated that the known driving circuits can be arranged in a way so as to connect to ground the diaphragm, as in fig.6a , with the rule that V_1 and V_2 have to be made superimposing V_{sig} out-of-phase to the same signs V_{pol} voltages or by V_{sig} in-phase superimposed to opposite signs V_{pol} .

The single sheet conductive diaphragm of fig.6a can be figured as two conductive plates electrically connected together and linked mechanically by a rigid insulating medium as in fig.6b, calling this arrangement “*bipolar diaphragm*” [5], which defines a four terminals ESL cell (fig.6c).

Connecting this ESL cell to a driving circuit according to the grounded diaphragm scheme, it is possible to swap the stators with the two plates of the diaphragm obtaining the circuit of fig.6d in which **the stators are grounded**.



To obtain an useful force on the diaphragm it is necessary to connect the diaphragm plates according to tab.2 , thus

$$V_1 = -V_{sig} \pm V_{pol} \quad \text{and} \quad V_2 = +V_{sig} \pm V_{pol} \quad (12)$$

$$\text{or} \quad V_1 = \pm V_{sig} - V_{pol} \quad \text{and} \quad V_2 = \pm V_{sig} + V_{pol} \quad (13)$$

In the first case the voltage applied between the diaphragm plates is given by

$$V_{diaph} = V_1 - V_2 = -V_{sig} \pm V_{pol} - V_{sig} \mp V_{pol} = -2V_{sig} \quad (14)$$

that gives rise to a short circuit current on the diaphragm plates capacitor overloading the driving circuit and thus producing a very low sound level affected by severe distortion.

In the second case the voltage applied to the diaphragm plates equivalent capacitor is due to

$$V_{diaph} = V_1 - V_2 = \pm V_{sig} - V_{pol} \mp V_{sig} - V_{pol} = -2V_{pol} \quad (15)$$

which is a static DC voltage without any variable component related to the audio signal that develops a static force inside the rigid insulating material of the bipolar diaphragm, as a mechanical spring loaded between two opposite inner walls of rigid box has no effect on any external force applied to the external faces of the box.

In this case the differential outputs of the driving circuit are loaded only by the capacitors between each stator and its corresponding diaphragm plate.

This is, as depicted in fig.7, the arrangement that develops the sound in a grounded stators ESL cell.

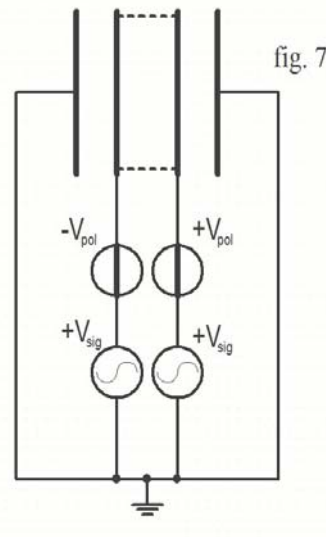
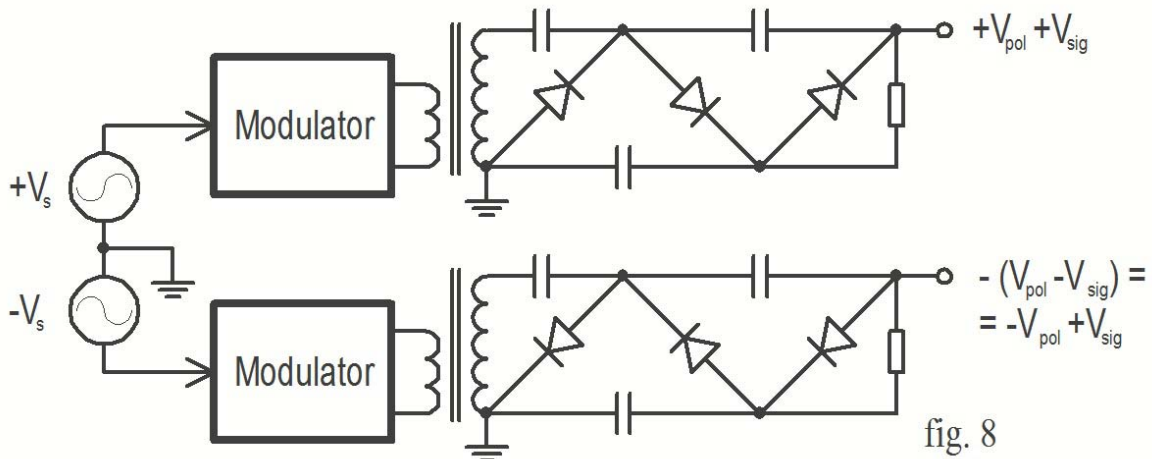


Fig.8 shows the block diagram of a suitable switching driving circuit based on Villard's voltage multipliers; in this circuit load resistors have to be employed to avoid that the high voltage outputs lock to the maximum peak value as happens in a clamp circuit.



A possible way to make a bipolar diaphragm

The classic two parallel-plates capacitor equation (1) and consequently the general purpose ESL equation (2) work with the hypothesis of having the gap thickness d constant.

It's well known that this condition is obtained in a classic ESL cell making the diaphragm with a very thin layer held in place on its surrounding frame by a very strong mechanical tension; this layer can vibrate under the effect of F_{diaph} but the variations around the nominal value of d are negligible.

A rigid but vibrating bipolar diaphragm can be made using a common two layers printed circuit board, choosing the thinnest thickness available (ie. 0.2 .. 0.4 mm) for the dielectric material and the thinnest thickness available (ie. 0.5 oz, 17.5 μm) for the copper.

The two copper sides can be etched with any kind of interleaved pattern in order to minimize the capacitance between the two layers to avoid a capacitive overload on the outputs of the differential amplifier that feeds the ESL cell.

This rigid bipolar diaphragm can be made vibrating by some hinges engraved by slots between the frame part and the central vibrating surface as in fig.9, where the view of one slotted hinge is enlarged to show it in detail.

An experimental bipolar diaphragm was crafted with the size of an A4 sheet on 0.4 mm FR4 substrate, obtaining 2880 pF between its plates and 496 pF between each diaphragm plate and the corresponding stator made by a square perforated steel sheet and mounted over with a 1.6 mm gap.

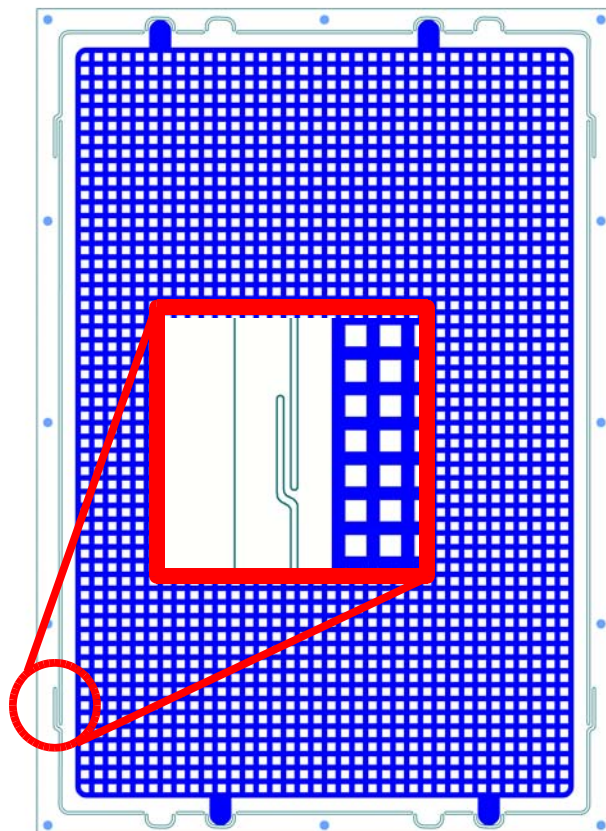


fig.9

Further ideas to investigate

1) According to the third Newton's law of motion, if the diaphragm is subject to a force F_{diaph} , the forces F_{stat1} and F_{stat2} are developed with the same intensity but reversed sign on the stators

$$\begin{aligned} F_{diaph} + F_{stat1} &= 0 \Rightarrow F_{diaph} = -F_{stat1} \\ F_{diaph} + F_{stat2} &= 0 \Rightarrow F_{diaph} = -F_{stat2} \end{aligned} \quad (16)$$

That means an ESL cell can be developed with a static bipolar diaphragm and two vibrating grounded stators.

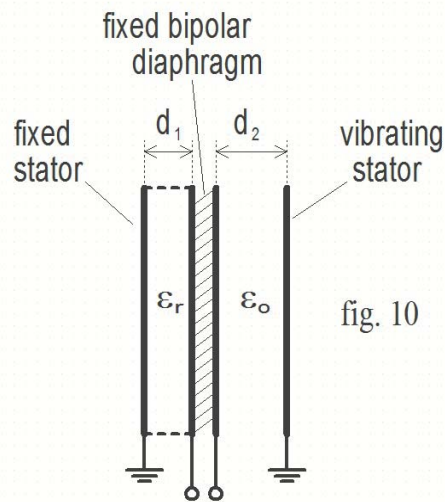
2) Further, both the forces F_{diaph} and F_{stat1} (or F_{stat2}) can be developed without any motion of their related plates or variation of the gap d , as a brick lying on a table is statically subject to the gravity force without any vertical displacement.

That means an ESL cell can be developed with only one grounded vibrating stator, having the other grounded stator glued to the rigid, motionless bipolar diaphragm.

3) The last proposal regards an ESL cell with asymmetrical gaps $d_1 \neq d_2$ filled with different dielectrics, being ϵ_r the relative dielectric permittivity of the media between the first grounded stator and the bipolar diaphragm as in fig.10 .

In order to cancel the quadratic terms in (2) it is necessary to keep

$$\frac{\epsilon_r \epsilon_0 A}{2 d_1^2} = \frac{\epsilon_0 A}{2 d_2^2} \quad (17) \quad \text{that means} \quad d_1 = \sqrt{\epsilon_r} d_2 \quad (18)$$



References

- [1] M.Moresco, M.Nigro, “Complementi di fisica generale”, Cleup Editore, 1985, pg. 60-63
- [2] P.J.Baxandall in J.Borwick ed., “Loudspeaker and Headphone Handbook”, Focal Press, 2001, (eq. 3.12 at pg.112)
- [3] Acoustat Corp., “The Acoustat Servo-Charge Amplifier Service and Owner's Manual”, 1979 (?), pg. 19
- [4] Final Sound Solutions, “Final Inverter Technology for Electrostatic Speakers”, white paper, 2005, pg. 4
- [5] in “Distortion in Electrostatic Loudspeakers”, Wireless World, Feb. 1956, pg.54-55 the unknown author proposed a bipolar diaphragm with the two conducting surfaces separately fed through high resistances in an attempt to minimize some distortions. But in this case the two stators are not explicitly connected to ground.