AmpDiVa White Paper

On a general purpose equation for three terminals ESL cells and following improvements about them

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date of first issue May 10th, 2023

The famous Walker's equation relates the sound pressure diffused by a planar ESL to the electrostatic force developed on its diaphragm, being this force due to the voltage applied to the cell plates.
The equation that relates the diaphragm force to the cell voltage has been historically evaluated starting from the classic driving circuit made with a center tap transformer connected to the cell stators and an HV bias generator tied to the diaphragm.
A general purpose force equation indipendent from the kind of the driving circuit is depicted and then evaluated in a new class of ESL cells with *bipolar diaphragm, grounded stators* and *asymmetrical structure*.

Prologue

The P. J. Baxandall's chapter on ESL reports in [1, (3.87), pg. 156] the Walker's equation as the sound pressure P at the distance of listening r obtained by a force F_{sig} applied to a vibrating diaphragm at frequency f

$$P = \frac{F_{sig}\, f}{2\, c\, r}$$

where c is the speed of the sound in air

and F_{sig} is due to the total audio signal amplitude voltage V_{sig} applied to the stators and the constant bias (or polarizing) voltage V_{pol} applied to the diaphragm of a three plates planar ESL cell of area A and gap d

$$F_{sig} = \frac{\varepsilon_0 A V_{pol} V_{sig}}{d^2} \qquad [1, (3.15), pg. 112]$$

where $\varepsilon_0 = 8.854 \cdot 10^{-12}$ F/m is the absolute dielectric permittivity (dielectric constant) of the air, which is very close to that of the vacuum.

In [1] this F_{sig} equation has been developed by a tangled process starting from the classic ESL driving circuit based on a center tap transformer ([1], fig.3.1b, pg. 109) and with the assumption of a constant charge $+Q_D$ on the diaphragm.

In the following, it will be shown that the equation for F_{sig} can be developed simply knowing the voltage differences applied between the stators and the diaphragm on the three terminals of a classic three plates ESL cell.

General purpose ESL diaphragm force equation

The force equation between the plates of an ideal *parallel plates capacitor* can be found in [1] (3.1, pg. 109) or in other physics text books, for example in [2] where it has been developed for an insulated capacitor previously charged with a Q charge and for a capacitor tied to a voltage generator V, giving always the same result for both the cases

$$F = \frac{\varepsilon_0 A V^2}{2 d^2} \qquad (1)$$

The square on the voltage in this equation tells that the force F developed by a parallel plates capacitor can not be used as an audio source as the relationship with the voltage is not linear, but also tells that the force F is always attractive towards the plates.

As a classic ESL cell can be seen as two parallel plates capacitors with one common plate (the diaphragm), the force F_{diaph} developed on this common plate is simply given by the difference between the two actractive forces F_1 between the first stator and the diaphragm and F_2 between the diaphragm and the second stator

$$F_{diaph} = F_1 - F_2 = \frac{\varepsilon_0 A V_1^2}{2 d^2} - \frac{\varepsilon_0 A V_2^2}{2 d^2} = \frac{\varepsilon_0 A}{2 d^2}(V_1^2 - V_2^2) \qquad (2)$$

where the voltages V_1 and V_2 applied between the plates can be written as the voltage differences between the voltages on the three ESL cell terminals and a common reference, for example the ground potential (fig. 1)

$$V_1 = V_{stat1} - V_{diaph}$$
$$V_2 = V_{stat2} - V_{diaph} \qquad (3)$$

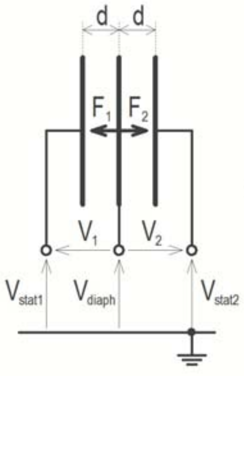


fig. 1

Evaluation of the general purpose equation with known ESL cell driving method

1) When the ESL cell is driven by the classic center tap transformer tied to the stators (the so called "push-pull" method) with the diaphragm biased by a fixed high voltage generator (fig. 2), we have

$$V_{\text{stat1}} = -V_{sig} \qquad V_{\text{diaph}} = V_{\text{pol}} \qquad V_{\text{stat2}} = +V_{\text{sig}}$$

therefore $$F_{\text{diaph}} = \frac{\varepsilon_0 A}{2d^2}\left[\left(\left(-V_{sig}\right)-V_{pol}\right)^2-\left(\left(+V_{sig}\right)-V_{pol}\right)^2\right] =$$

$$= \frac{\varepsilon_0 A}{2d^2}\left[V_{sig}^2+2\text{V}_{sig}V_{pol}+Vpol^2-\left(V_{sig}^2-2\text{V}_{sig}V_{pol}+V_{pol}^2\right)\right] =$$

$$= \frac{\varepsilon_0 A}{2d^2}\left[V_{sig}^2+2\text{V}_{sig}V_{pol}+Vpol^2-V_{sig}^2+2\text{V}_{sig}V_{pol}-V_{pol}^2\right] =$$

$$= \frac{\varepsilon_0 A}{2d^2}4\text{V}_{sig}V_{pol} = 2\frac{\varepsilon_0 A}{d^2}V_{sig}V_{pol} \qquad (4)$$

that is exactly the F_{sig} equation (3.15) in [1], except for the multiplier ***2*** that is due to the $\pm V_{sig}$ definition used in fig.2 instead of the $\pm\frac{V_{sig}}{2}$ used on the original development of the (3.15).
Note that V_{pol} can be seen as a gain term for F_{diaph} as it is know that it can be used as a volume control for the ESL cell; it can be also negative and in that case F_{diaph} becames also negative.
This doesn't mean that with $V_{\text{pol}}<0$ the F_{diaph} is a repulsive force but it is only in phase opposition to the V_{sig}.

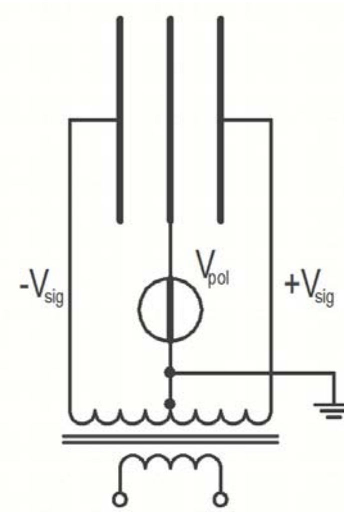


fig. 2

2) If the ESL cell is fed by an SRPP (or a cascode) linear tube amplifier with differential outputs provided by a single power supply rail like the Acoustat Servo-Charge Amplifier [3] or other more modern similar linear circuit based on semiconductors (fig. 3), we have

$$V_{\text{stat1}} = -V_{sig}+V_Q \qquad V_{\text{diaph}} = V_{\text{pol}} \qquad V_{\text{stat2}} = +V_{\text{sig}}+V_{\text{Q}}$$

where V_{Q} is the quiescent point voltage at the amplifier outputs when the signal V_{sig} is null, thus

$$F_{\text{diaph}} = \frac{\varepsilon_0 A}{2d^2}\left[\left((-V_{sig}+V_Q)-V_{pol}\right)^2-\left((+V_{sig}+V_Q)-V_{pol}\right)^2\right] = \dots$$

$$\dots = 2\frac{\varepsilon_0 A}{d^2}V_{sig}(V_{pol}-V_Q) \quad (5)$$

In this case the gain term ($V_{pol} - V_Q$) is reduced by the quiescent point voltage V_Q, as this is typically positive due to the circuit adopted. The gain can be raised if V_{pol} is negative and in this case due to the effect of the addition of V_Q, the same gain of eq. (4) can be reached with a lower value of V_{pol}.
If the amplifier outputs is AC coupled to the stators with capacitors, V_Q becames 0 and equation (5) becames equal to equation (4).
On another hand, if V_Q is large enough to allow the sound reproduction, V_{pol} can be set to 0 leading to an <u>ESL cell with the diaphragm grounded</u>.

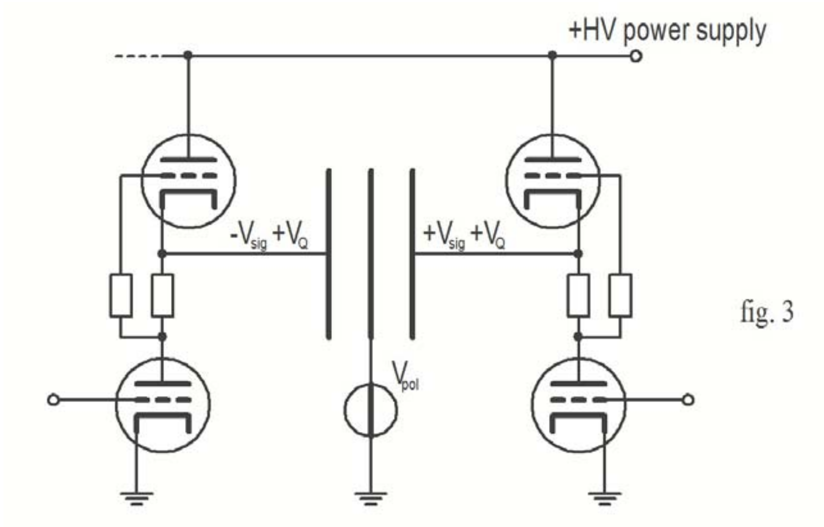


fig. 3

3) A different approach has been proposed by Final Sound Solutions in their white paper [4]: the “Inverted ESL” is a circuit (fig. 4) with a common mode V_{sig} driving the diaphragm and the stators biased at HV of opposite sign

$$V_{\text{stat1}} = -V_{pol} \qquad V_{\text{diaph}} = V_{\text{sig}} \qquad V_{\text{stat2}} = +V_{\text{pol}}$$

and again we have $$F_{\text{diaph}} = \frac{\varepsilon_0 A}{2d^2}\left[(-V_{pol}-V_{sig})^2-(+V_{pol}-V_{sig})^2\right] = \dots = 2\frac{\varepsilon_0 A}{d^2}V_{sig}V_{pol} \quad (6)$$

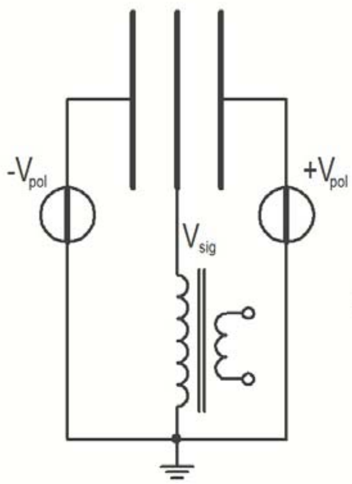


fig. 4

Evaluation of the general purpose equation in case of a grounded stators ESL cell

It's quite trivial to show that the previous circuit of fig. 4 is equivalent to the grounded diaphragm circuit depicted in fig.5, where

$$V_{\text{stat1}} = -V_{pol} - V_{sig} \qquad V_{\text{diaph}} = 0 \qquad V_{\text{stat2}} = +V_{pol} - V_{sig}$$

ans thus also in this case we have again the same result of eq. (6).

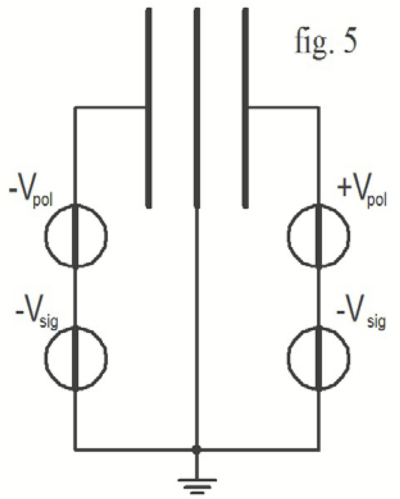


fig. 5

Note that in fig. 5 $V_{\text{stat1}} = -V_{\text{pol}} - V_{\text{sig}} = -(V_{\text{pol}} + V_{\text{sig}})$

means V_{stat1} and V_{stat2} could be obtained for example making a pair of amplitude modulated high frequency high voltage generators controlled by the $\pm V_s$ 180° out of phase signals and followed by Villard's voltage multipliers as suggested by fig. 6.
In this circuit the load resistors have to be employed to avoid that the high voltage outputs lock to the maximum peak value as happens in a clamp circuit.

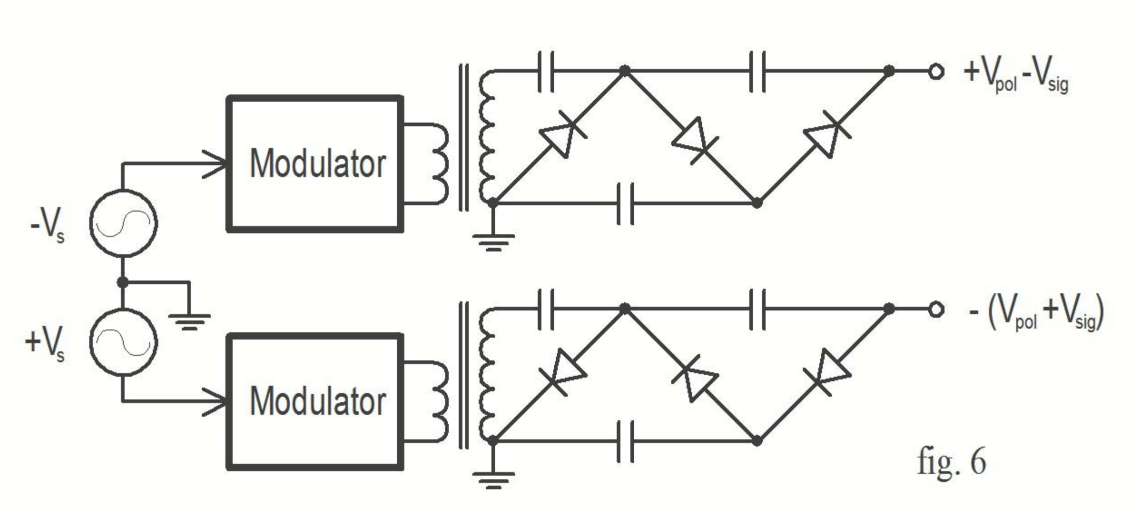
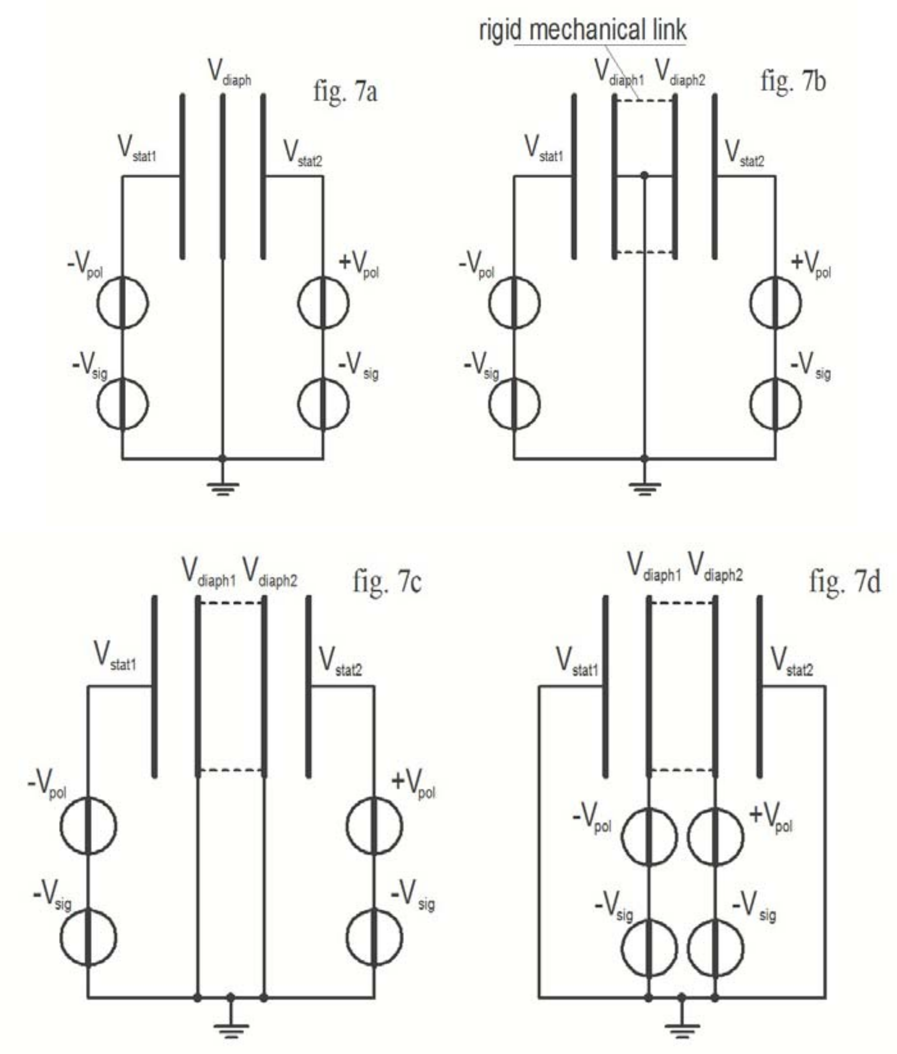


fig. 6

With reference to the grounded diaphragm circuit of fig.5 simplified in fig.7a, we can now figure the diaphragm made as two plates electrically connected togheter and linked mechanically by a rigid insulating medium as in fig.7b, calling this arrangement as "***bipolar diaphragm***". If we tie this ESL cell to a circuit with the grounded diaphragm scheme, it is possible to swap the stators with the two plates of the diaphragm obtaining the circuit of fig.7d in which **the stators are grounded**.



In this case the force equation can be written again as in (2) where

$$\begin{aligned} V_1 &= V_{stat1} - V_{diaph1} = 0 - (V_{pol} + V_{sig}) \\ V_2 &= V_{stat2} - V_{diaph} = 0 - (V_{pol} - V_{sig}) \end{aligned} \qquad (7)$$

giving always $$F_{\text{diaph}} = 2\frac{\varepsilon_0 A}{d^2} V_{sig} V_{pol}$$

Between the plates of the bipolar diaphragm an attractive force F_{dd} is developed according to the equation (1)

$$F_{\text{dd}} = \frac{\varepsilon_0 A V^2}{2 d_{bd}^2} = \frac{\varepsilon_0 A(-V_{pol} - V_{sig} - (+V_{pol} - V_{sig}))^2}{2 d_{bd}^2} = \frac{2\varepsilon_0 A V_{pol}^2}{d_{bd}^2}$$

that is a static force related only to V_{pol}.

F_{dd} no any effect on the F_{diaph} as it is developed inside the rigid insulating material (of ε_r relative dielectric permittivity and thickness d_{bd}) that made the bipolar diaphragm, as a mechanical spring loaded between two opposite inner walls of rigid box, has no effect on any external force applied to the external faces of the box.

A possible way to make a bipolar diaphragm

The classic two parallel-plates capacitor equation (1) ad consequently the general purpose ESL equation (2) work with the hypothesis of having the gap tickness *d* constant.

It's well known that this condition is obtained in a classic ESL cell making the diaphragm with a very thin layer held in place on its surrounding frame by a very strong mechanical tension; this layer can vibrate under the effect of F_{diaph} but the variations around the nominal value of *d* are negligible.

A rigid but vibrating bipolar diaphragm can be made using a common two layers printed circuit board, choosing the thinnest thickness available (ie. 0.2 mm) for the dielectric material and the thinnest thickness available (ie. 0.5 oz, 17.5 µm) for the copper.
The two copper sides can be etched with a pattern that minimize the capacitance between the two layers of this diaphragm in order to avoid a capacitive overload on the outputs of the differential amplifier that feeds the ESL cell.
This rigid bipolar diaphragm can be made vibrating by some hinges engraved by slots between the frame part and the central vibrating surface as in fig. 8, where the view of one slotted hinge is enlarged to show it in detail.

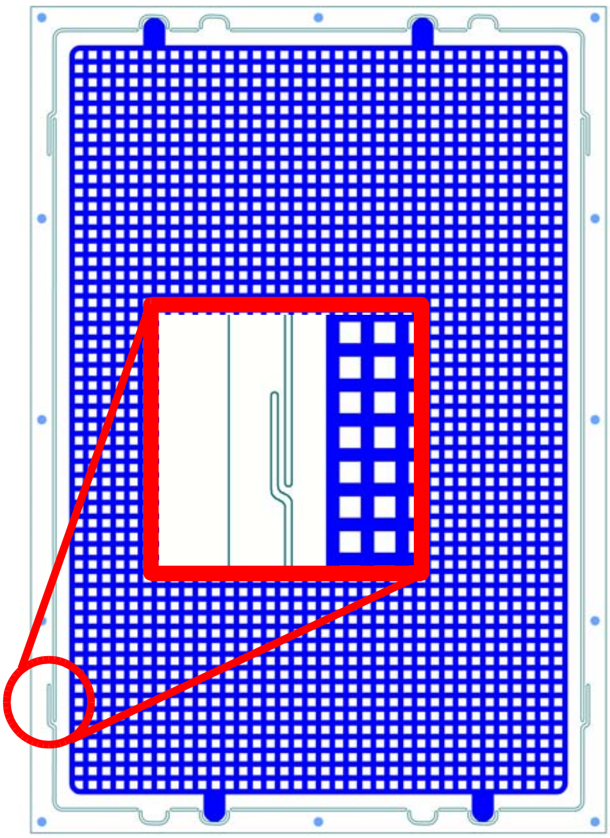


fig.8

Further ideas to investigate

1) According to the third Newton's law of motion, if the diaphragm is subject to a force F_{diaph}, the forces F_{stat1} and F_{stat2} are developed with the same intensity but reversed sign on the stators

$$\begin{aligned} F_{diaph} + F_{stat1} &= 0 \quad \Rightarrow \quad F_{diaph} = -F_{stat1} \\ F_{diaph} + F_{stat2} &= 0 \quad \Rightarrow \quad F_{diaph} = -F_{stat2} \end{aligned} \tag{8}$$

That means an ESL cell can be developed with a static bipolar diaphragm and two vibrating grounded stators.

2) Further, both the forces F_{diaph} and F_{stat1} (or F_{stat2}) can be developed without any motion of their related plates or variation of the gap *d*, as a brick lying on a table is statically subject to the gravity force without any vertical displacement.
That means an ESL cell can be developed with only one grounded vibrating stator, having the other grounded stator glued to the rigid, motionless bipolar diaphragm.

3) The last improvement can be figured writing the equation (2) in the case of an asymmetrical ESL cell, in which there is a dielectric material that fills the gap d_1 between the first grounded stator and the bipolar diaphragm, leaving an air gap d_2 between the other side of the diaphragm and the second grounded stator as in fig. 9.

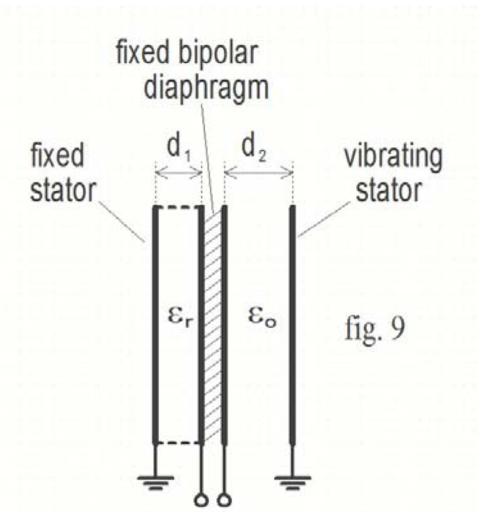


fig. 9

In this case the equation (2) is given by

$$F_{\text{stat2}} = -F_{diaph} = -(F_1 - F_2) = \frac{\varepsilon_0 A V_2^2}{2 d_1^2} - \frac{\varepsilon_r \varepsilon_0 A V_1^2}{2 d_2^2} \tag{9}$$

being ε_r the relative dielectric permittivity of the media between the first grounded stator and the bipolar diaphragm.

If we equals $\dfrac{\varepsilon_0}{d_1^2} = \dfrac{\varepsilon_r \varepsilon_0}{d_2^2}$

that means $d_2 = \sqrt{\varepsilon_r}\, d_1$ (10)

we obtain the usual result $F_{\text{stat2}} = \; -F_{diaph} \; = \; -2\frac{\varepsilon_0 A}{d_1^2} V_{sig} V_{pol}$

even if it is related to an asymmetrical ESL cell with grounded stators and a single vibrating one.

References

[1] J.Borwick ed., “Loudspeaker and Headphone Handbook”, Focal Press, 2001
[2] M.Moresco, M.Nigro, “Complementi di fisica generale”, Cleup Editore, 1985, pg. 60-63
[3] Acoustat Corp., “The Acoustat Servo-Charge Amplifier Service and Owner's Manual”, 1979 (?), pg. 19
[4] Final Sound Solutions, “Final Inverter Technology for Electrostatic Speakers”, white paper, 2005, pg. 4